

2.1 Conditional Statements

- Goals**
- Recognize and analyze a conditional statement.
 - Write postulates about points, lines, and planes using conditional statements.

VOCABULARY

Conditional statement

If-then form

Hypothesis

Conclusion

Converse

Negation

Inverse

Contrapositive

Equivalent statements

Example 1 *Rewriting in If-Then Form*

Rewrite the conditional statement in *if-then* form.

- a. Three points are coplanar if they lie on the same plane.
- b. Water freezes at temperatures below 32°F .
- c. An even number is divisible by 2.

Solution

- a. If _____, then _____.
- b. If _____, then _____.
- c. If _____, then _____.

Example 2 *Writing an Inverse, Converse, and Contrapositive*

Write the (a) inverse, (b) converse, and (c) contrapositive of the following statement.

If the sun is shining, then we are not watching TV.

Solution

- ~~a. Inverse:~~ _____.
- b. Converse: _____.
- ~~c. Contrapositive:~~ _____.

✓ **Checkpoint** Write the ~~(a) inverse~~, (b) converse, and ~~(c) contrapositive of the conditional statement.~~

1. If my allowance increases, then I can save more money.

POINT, LINE, AND PLANE POSTULATES

Postulate 5 Through any two points there exists exactly one _____.

Postulate 6 A line contains at least two _____.

Postulate 7 If two lines intersect, then their intersection is _____.

Postulate 8 Through any three _____ points there exists exactly one plane.

Postulate 9 A plane contains at least three _____ points.

Postulate 10 If two points lie in a plane, then the line containing them _____.

Postulate 11 If two planes intersect, then their intersection is a _____.

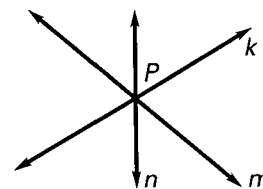
Example 3 *Using Postulates and Counterexamples*

Decide whether the statement is *true* or *false*. If it is false, give a counterexample.

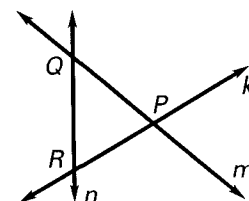
- A point can lie on more than two lines.
- Three lines can intersect at no more than three distinct points.
- If two lines are coplanar, then they intersect.

Solution

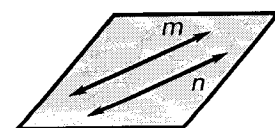
a. In the diagram at the right, point P is the _____ of line k , line m , and line n . So, it is _____ that a point can lie on more than two lines.



b. In the diagram at the right, line k and line m intersect at point _____, line _____ and line _____ intersect at point Q , and line _____ and line n intersect at point _____. There are no more possible intersections. So, it is _____ that three lines can intersect at no more than three distinct points.



c. In the diagram at the right, line m and line n are _____, but they do not _____. So, it is _____ that if two lines are coplanar, then they intersect.



Study Guide

2.1 Blue Book

Conditional Statements and Their Converses

If-then statements are commonly used in everyday life. For example, an advertisement might say, "If you buy our product, then you will be happy." Notice that an if-then statement has two parts, a *hypothesis* (the part following "if") and a *conclusion* (the part following "then").

New statements can be formed from the original statement.

Statement	$p \rightarrow q$
Converse	$q \rightarrow p$

Example: Rewrite the following statement in if-then form. Then write the converse, inverse, and contrapositive.

All elephants are mammals.

If-then form: If an animal is an elephant, then it is a mammal.

Converse: If an animal is a mammal, then it is an elephant.

Identify the hypothesis and conclusion of each conditional statement.

1. If today is Monday, then tomorrow is Tuesday.
2. If a truck weighs 2 tons, then it weighs 4000 pounds.

Write each conditional statement in if-then form.

3. All chimpanzees love bananas.
4. Collinear points lie on the same line.

Write the converse of each conditional.

5. If an animal is a fish, then it can swim.
6. All right angles are congruent.

Conditional Statements and Their Converses

Identify the hypothesis and the conclusion of each statement.

1. If you purchase a computer and do not like it, then you can return it within 30 days.
2. If $x + 8 = 15$, then $x = 7$
3. If the drama club raises \$2000, then they will go on tour.
4. If the temperature today is 80° or more, then you will go swimming.
5. If two lines intersect, then the intersection is a point.

the converse
Write ~~two other forms~~ of each statement.

6. If two planes intersect, then the intersection is a line.
7. If it snows, then you will go sledding.
8. Your dog will be happy if you feed him Doggy Chow.
9. Hiking will be easier if you have hiking boots.
10. All squares have four sides of equal length and four right angles..

Write the converse of each statement.

11. If a figure is a triangle, then it has three sides.
12. If you find a penny, then you will have good luck.
13. If you ride your bicycle recklessly, then you can get hurt.
14. If two distinct lines intersect, then their intersection is one point.
15. If your cat purrs, then it is contented.

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Practice A

For use with pages 71–78

Lesson 2.1

Identify the hypothesis and the conclusion.

1. If the weather is warm, then we should go swimming.
2. If you want good service, then take your car to Joe's Service Center.
3. If you like purple, you'll love this sweater.
4. $2x - 12 = 40$ only if $x = 26$.
5. If the groundhog sees its shadow, then there will be six more weeks of winter.

Rewrite the conditional statement in if-then form.

6. Today is Monday if yesterday was Sunday.
7. An object measures 12 inches if it is one foot long.
8. A number is divisible by 4 if it is divisible by 8.
9. An acute angle is an angle that measures less than 90° .
10. Geometry is offered only during periods 2 and 4.

Decide whether the statement is *true* or *false*. If false, provide a counterexample.

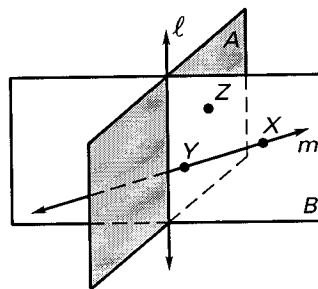
11. The equation $2x - 7 = 5 + x$ has exactly one solution.
12. If $x^2 = 16$, then x must equal 8 or -8 .
13. February 14 is Valentine's Day.
14. If you visited the Statue of Liberty, then you've been to New York.
15. A point may lie on at most two lines.

Write the converse and contrapositive of each statement.

16. If you like tennis, then you play on the tennis team.
17. If x is odd, then $2x$ is even.
18. If $m\angle P = 45^\circ$, then $\angle P$ is acute.

Use the diagram to state the postulate(s) that verifies the truth of the statement.

19. The points X , Y , and Z lie in a plane (labeled B).
20. The points X and Y lie on a line (labeled m).
21. The planes A and B intersect in a line (labeled l).
22. The points X and Y lie in a plane B . Therefore, line m lies in plane B .



2.2

Definitions and Biconditional Statements

54c

- Goals**
- Recognize and use definitions.
 - Recognize and use biconditional statements.

VOCABULARY

Perpendicular lines

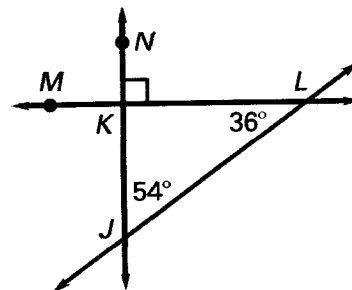
Line perpendicular to a plane

Biconditional statement

Example 1 Using Definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

- $\angle KLJ$ and $\angle KJL$ are complementary.
- \overleftrightarrow{KL} and \overleftrightarrow{LJ} are perpendicular.
- $\angle MKJ$ is a right angle.



Solution

- This statement is true. Two angles are complementary if the sum of their measures is 90. $m\angle KLJ + m\angle KJL = 90$, so the angles are complementary.
- This statement is false. \overleftrightarrow{KL} and \overleftrightarrow{LJ} do not intersect to form a right angle. So, the lines are not perpendicular.
- This statement is true. $\angle MKJ$ and $\angle NKL$ are vertical angles. $\angle NKL$ is a right angle. Because vertical angles are congruent, $\angle MKJ$ is a right angle.

- 54a
- ✓ **Checkpoint** Use the diagram in Example 1 to decide whether the statement is true. Explain your answer using the definitions you have learned.

1. $\angle KJL$ is an acute angle.	2. Point N is in the interior of $\angle KLJ$.
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Example 2 *Rewriting a Biconditional Statement*

Rewrite the following biconditional statement as a conditional statement and its converse.

An angle is a straight angle if and only if its measure is 180° .

Conditional statement: If _____, then _____.

Converse: If _____, then _____.

Example 3 *Analyzing a Biconditional Statement*

Consider the following statement: $x = 2$ if and only if $3x + 5x = 10x - 2x$.

- a. Is this a biconditional statement? b. Is the statement true?

Solution

a. The statement is biconditional because it contains the phrase _____.

b. The statement can be rewritten as the following statement and its converse.

Conditional statement: If _____, then _____.

Converse: If _____, then _____.

The first statement is _____. The second statement is _____. So, the biconditional statement is _____.

Are there any values other than $x = 2$ that make the equation true?

Each of the following statements is true. Write the converse of each statement and decide whether the converse is *true* or *false*. If the converse is true, combine it with the original statement to form a true biconditional statement. If the converse is false, state a counterexample.

a. If $\sqrt{x} = 1$, then $x = 1$.

b. If two angles are vertical angles, then they are congruent.

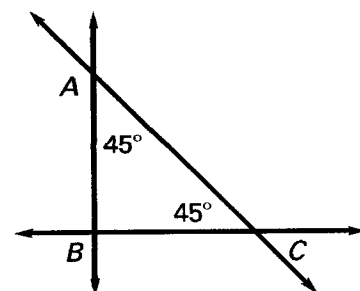
Solution

a. Converse: _____. The converse is _____.

Biconditional statement: _____

b. Converse: _____

_____. The converse is _____. As a
_____, consider the figure
at the right. \angle _____ and \angle _____ are
congruent, but they are not _____
_____.



✓ Checkpoint Complete the following exercises.

3. Rewrite the following biconditional statement as a conditional statement and its converse.

Two angles are supplementary if and only if the sum of their measures is 180° .

4. Consider the following statement: Two segments are congruent if and only if they have the same length.

a. Is the statement biconditional?

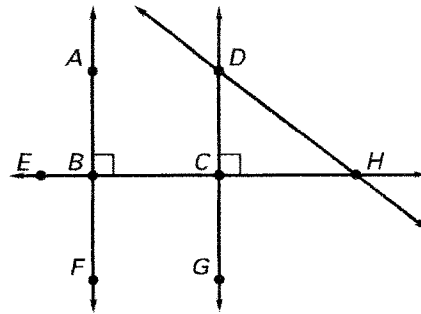
b. Is the statement *true* or *false*?

Practice A

For use with pages 79–85

Use the diagram to determine whether the statement is **true** or **false**.

- Points A , B , and C are collinear.
- $\angle DCB$ and $\angle DCH$ are supplementary.
- Points E , D , and H lie in the same plane.
- \overleftrightarrow{DH} is perpendicular to \overleftrightarrow{EH} .
- \overleftrightarrow{HE} is perpendicular to \overleftrightarrow{AF} .
- $\angle DCB$ and $\angle ABC$ are complementary.
- Point C is the midpoint of \overline{BH} .



Rewrite the biconditional statement as a conditional statement and its converse.

- Two segments are congruent if and only if they have the same measure.
- Three points are collinear if and only if they lie on the same line.
- Four points are coplanar if and only if they lie in the same plane.
- You may go to the movies Friday night if and only if you clean your room.
- You may become president of the United States if and only if you are 35 years old.

Give a counterexample that demonstrates that the converse of the statement is false.

- If you live in Detroit, then you live in Michigan.
- If an angle measures 30° , then it is acute.
- If an animal is a leopard, then it has spots.
- If the month is September, then there are 30 days in the month.
- If two angles are vertical angles, then they are not adjacent.

In Exercises 18 and 19, use the information in the table to write a definition for each type of saxophone. The first one is started for you.

Instrument	Frequency (cycles per second)	
	Lower limit	Upper limit
E-flat baritone saxophone	69	416
B-flat tenor saxophone	104	622
E-flat alto saxophone	138	831

Sample: A saxophone that has a frequency of 69 cycles per second to 416 cycles per second is called an E-flat baritone saxophone.

- B-flat tenor saxophone
- E-flat alto saxophone

2.4 Reasoning with Properties from Algebra

- Goals**
- Use properties from algebra.
 - Use properties of length and measure to justify segment and angle relationships.

ALGEBRAIC PROPERTIES OF EQUALITY

Let a , b , and c be real numbers.

Addition Property If $a = b$, then $a + c = b + c$.

Subtraction Property If $a = b$, then $a - c = b - c$.

Multiplication Property If $a = b$, then $ac = bc$.

Division Property If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Reflexive Property For any real number a , $a = a$.

Symmetric Property If $a = b$, then $b = a$.

Transitive Property If $a = b$ and $b = c$, then $a = c$.

Substitution Property If $a = b$, then b can be substituted for a in any expression.

Example 1 Writing Reasons

Solve $-2x + 1 = 56 - 3x$ and write a reason for each step.

$$-2x + 1 = 56 - 3x \quad \text{Given}$$

$$\quad + 1 = 56$$

$$x =$$

Checkpoint Solve the equation. Write a reason for each step.

1. $12x - 3(x + 7) = 8x$

Solve

$$10y + 5 = 25$$

explain

PROPERTIES OF EQUALITY

	Segment Length	Angle Measure
Reflexive	For any segment AB , _____.	For any angle A , _____.
Symmetric	If $AB = CD$, then _____.	If $m\angle A = m\angle B$, then _____.
Transitive	If $AB = CD$ and $CD = EF$, then _____.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then _____.

Example 3

Using Properties of Measure

Use the information at the right to find $m\angle 1$.

$$\begin{aligned} m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 &= 360^\circ \\ m\angle 2 + m\angle 3 &= m\angle 4 \\ m\angle 1 &= m\angle 4 \end{aligned}$$

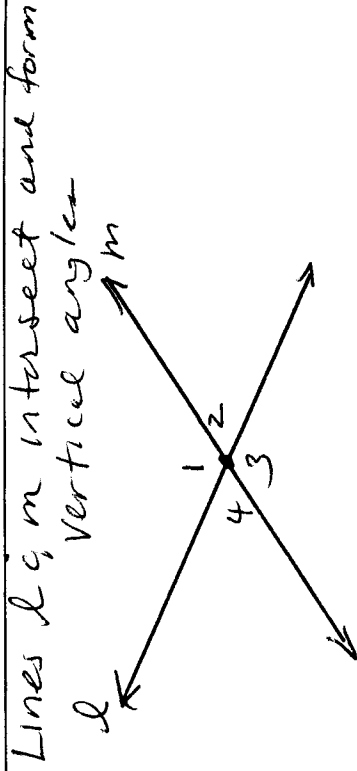
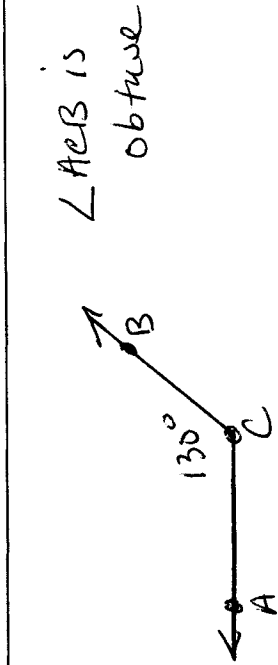
Solution

$$\begin{aligned} m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 &= \underline{\hspace{2cm}} \\ m\angle 2 + m\angle 3 &= \underline{\hspace{2cm}} \\ m\angle 1 &= \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} &= 360^\circ \\ 3(\underline{\hspace{2cm}}) &= 360^\circ \\ \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \\ m\angle 1 &= \underline{\hspace{2cm}} \end{aligned}$$

Given
Given
Given
Substitution property
of equality
Simplify.
Division property
of equality
Transitive property
of equality

Name _____

Period _____



Conditional Statement

Conditional Statement

Converse

Converse

Are Conditional and Converse True? If yes, write the conditional. If no, provide a counter example.
~~Conditional~~ Statement

Are Conditional and Converse True? If yes, write the conditional. If no, provide a counter example.
~~Conditional~~ Statement

Name _____ Period _____

<p>$\angle 1$ & $\angle 2$ are a linear pair</p>	<p>$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$</p>
<p>Conditional Statement</p>	<p>Conditional Statement</p>
<p>Converse</p>	<p>Converse</p>
<p>Are Conditional and Converse True? <i>If yes, write the biconditional. If no, provide a counter example.</i></p>	<p>Are Conditional and Converse True? <i>If yes, write the biconditional. If no, provide a counter example.</i></p>

ALGEBRAIC PROPERTIES OF EQUALITY

Let a , b , and c be real numbers.

ADDITION PROPERTY	If $a = b$, then $a + c = b + c$.
SUBTRACTION PROPERTY	If $a = b$, then $a - c = b - c$.
MULTIPLICATION PROPERTY	If $a = b$, then $ac = bc$.
DIVISION PROPERTY	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
REFLEXIVE PROPERTY	For any real number a , $a = a$.
SYMMETRIC PROPERTY	If $a = b$, then $b = a$.
TRANSITIVE PROPERTY	If $a = b$ and $b = c$, then $a = c$.
SUBSTITUTION PROPERTY	If $a = b$, then a can be substituted for b in any equation or expression.

Match the statement with the Property of Congruence.

- | | |
|--|------------------------|
| 1. For any segment \overline{XY} , $\overline{XY} \cong \overline{XY}$ | A. Transitive Property |
| 2. If $\overline{JK} \cong \overline{MN}$ and $\overline{MN} \cong \overline{CD}$, then $\overline{JK} \cong \overline{CD}$. | B. Symmetric Property |
| 3. If $\overline{BN} \cong \overline{TR}$, then $\overline{TR} \cong \overline{BN}$. | C. Reflexive Property |

Skill Check ✓ In Exercises 4–8, match the conditional statement with the property of equality.

- | | |
|--|----------------------------|
| 4. If $JK = PQ$ and $PQ = ST$, then $JK = ST$. | A. Addition property |
| 5. If $m\angle S = 30^\circ$, then $5^\circ + m\angle S = 35^\circ$. | B. Substitution property |
| 6. If $ST = 2$ and $SU = ST + 3$, then $SU = 5$. | C. Transitive property |
| 7. If $m\angle K = 45^\circ$, then $3(m\angle K) = 135^\circ$. | D. Symmetric property |
| 8. If $m\angle P = m\angle Q$, then $m\angle Q = m\angle P$. | E. Multiplication property |

Match the statement with the property.

- | | |
|--|--|
| 13. If $m\angle S = 45^\circ$, then $m\angle S + 45^\circ = 90^\circ$. | A. Symmetric property of equality |
| 14. If $UV = VW$, then $VW = UV$. | B. Multiplication property of equality |
| 15. If $AE = EG$ and $EG = JK$, then $AE = JK$. | C. Addition property of equality |
| 16. If $m\angle K = 9^\circ$, then $3(m\angle K) = 27^\circ$. | D. Transitive property of equality |

Match each statement with the appropriate property of equality.

1) If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

2) If $a = b$, then $a + c = b + c$.

3) If $a = b$, then $b = a$.

4) If $a = b$, then $a \cdot c = b \cdot c$.

5) If $a = b$, then $a - c = b - c$.

6) $a = a$

7) If $a = b$ and $b = c$, then $a = c$.

a) Addition Property

b) Subtraction Property

c) Multiplication Property

d) Division Property

e) Reflexive Property

f) Symmetric Property

g) Transitive Property

Justify each statement with a property from algebra or definition or postulate from geometry.

8) If $AB = CD$ and $BC = BC$, then $AB + BC = CD + BC$

9) If $3m\angle 1 = 93$ then $m\angle 1 = 31$

10) $\overline{BC} \cong \overline{BC}$

11) If point A is on the interior of $\angle XOY$, then $m\angle XOY = m\angle XOY$

12) If $m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 2$.

13) If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Practice A

For use with pages 96–101

Match the statement with the Property of Equality.

- | | |
|--|----------------------------|
| 1. If $JK = PQ$ and $PQ = ST$, then $JK = ST$. | A. Addition property |
| 2. If $m\angle S = 30^\circ$, then $5^\circ + m\angle S = 35^\circ$. | B. Reflexive property |
| 3. If $AB + CD = EF + CD$, then $AB = EF$. | C. Substitution property |
| 4. $AB = AB$ | D. Transitive property |
| 5. If $ST = 2$, then $ST + TU = 2 + TU$. | E. Symmetric property |
| 6. If $m\angle K = 45^\circ$, then $3(m\angle K) = 135^\circ$. | F. Multiplication property |
| 7. If $m\angle P = m\angle Q$, then $m\angle Q = m\angle P$. | G. Subtraction property |

In Exercises 8–13, use the property to complete the statement.

8. Addition property of equality: If $AB = 5$, then $10 + AB = \underline{\quad ? \quad}$.
9. Multiplication property of equality: If $m\angle C = 30^\circ$, then $\underline{\quad ? \quad} (m\angle C) = 15^\circ$.
10. Reflexive property of equality: $AF = \underline{\quad ? \quad}$.
11. Symmetric property of equality: If $m\angle DCF = m\angle MJC$, then $\underline{\quad ? \quad}$.
12. Transitive property of equality: If $YZ = DB$ and $\underline{\quad ? \quad} = JK$, then $\underline{\quad ? \quad}$.
13. Substitution property of equality: If $MN = 3$, then $5(MN) = \underline{\quad ? \quad}$.

Complete the argument, giving a reason for each step.

14. $3(2x - 4) = 5x + 2$ Given

$6x - 12 = 5x + 2$ a. $\underline{\quad ? \quad}$

$x - 12 = 2$ b. $\underline{\quad ? \quad}$

$x = 14$ c. $\underline{\quad ? \quad}$

15. $4x + 8 = 2x - 12$ Given

$2x + 8 = -12$ a. $\underline{\quad ? \quad}$

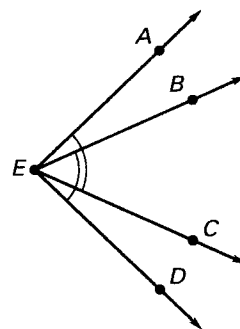
$2x = -20$ b. $\underline{\quad ? \quad}$

$x = -10$ c. $\underline{\quad ? \quad}$

16. $m\angle AEB + m\angle BEC = m\angle CED + m\angle BEC$ Given

$m\angle BEC = m\angle BEC$ a. $\underline{\quad ? \quad}$

$m\angle AEB = m\angle CED$ b. $\underline{\quad ? \quad}$

**In Exercises 17 and 18, solve the equation and state a reason for each step.**

17. $5(2x - 1) = 9x + 4$
18. $-4(x - 5) = 13$

Study Guide

Preparing for Two-Column Proofs

Many rules from algebra are used in geometry.

Properties of Equality for Real Numbers	
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

Example: Prove that if $4x - 8 = -8$, then $x = 0$.

Given: $4x - 8 = -8$

Prove: $x = 0$

Proof:

Statements	Reasons
a. $4x - 8 = -8$	a. Given
b. $4x = 0$	b. Addition Property (=)
c. $x = 0$	c. Division Property (=)

Name the property that justifies each statement.

1. Prove that if $\frac{3}{5}x = -9$, then $x = -15$.

Given: $\frac{3}{5}x = -9$

Prove: $x = -15$

Proof:

Statements	Reasons
a. $\frac{3}{5}x = -9$	a. _____
b. $3x = -45$	b. _____
c. $x = -15$	c. _____

2. Prove that if $3x - 2 = x - 8$, then $x = -3$.

Given: $3x - 2 = x - 8$

Prove: $x = -3$

Proof:

Statements	Reasons
a. $3x - 2 = x - 8$	a. _____
b. $2x - 2 = -8$	b. _____
c. $2x = -6$	c. _____
d. $x = -3$	d. _____

Practice**Preparing for Two-Column Proofs****Name the property or equality that justifies each statement.**

1. If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
2. If $x + 3 = 17$, then $x = 14$.
3. $xy = xy$
4. If $7x = 42$, then $x = 6$.
5. If $XY - YZ = XM$, then $XM + YZ = XY$.
6. $2(x + 4) = 2x + 8$
7. If $m\angle A + m\angle B = 90$, and $m\angle A = 30$, then $30 + m\angle B = 90$.
8. If $x = y + 3$ and $y + 3 = 10$, then $x = 10$.

Complete each proof by naming the property that justifies each statement.

9. Prove that if $2(x - 3) = 8$, then $x = 7$.

Given: $2(x - 3) = 8$ **Prove:** $x = 7$ **Proof:**

Statements	Reasons
a. $2(x - 3) = 8$	a. _____
b. $2x - 6 = 8$	b. _____
c. $2x = 14$	c. _____
d. $x = 7$	d. _____

10. Prove that if $3x - 4 = \frac{1}{2}x + 6$, then $x = 4$.

Given: $3x - 4 = \frac{1}{2}x + 6$ **Prove:** $x = 4$ **Proof:**

Statements	Reasons
a. $3x - 4 = \frac{1}{2}x + 6$	a. _____
b. $\frac{5}{2}x - 4 = 6$	b. _____
c. $\frac{5}{2}x = 10$	c. _____
d. $x = 4$	d. _____

Reading to Learn Mathematics

Preparing for Two-Column Proofs

Key Terms

two-column proof a deductive argument that contains statements and reasons organized in two columns.

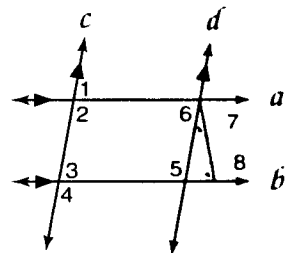
Reading the Lesson

- State whether each statement is *true* or *false*. If the statement is false, explain why.
 - Algebraic properties can be used as reasons in proofs.
 - When you solve an equation, you are using inductive reasoning.
 - In a two-column proof, you must give a reason for each statement.
 - The last statement in a two-column proof is the given information.

- Fill in the missing statements and reasons in the two-column proof.

Given: $a \parallel b$, $c \parallel d$

Prove: $m\angle 2 = m\angle 7 + m\angle 8$



Proof:

Statements

Reasons

a. _____	a. Given
b. $\angle 2 \cong \angle 4$	b. _____
c. $\angle 4 \cong \angle 5$	c. _____
d. _____	d. Transitive Property of Congruence
e. $m\angle 2 = m\angle 5$	e. _____
f. $m\angle 5 = m\angle 7 + m\angle 8$	f. _____
g. _____	g. Substitution Property of Equality

Helping You Remember

- A good way to remember some terms is to compare them. Write several sentences comparing the similarities and differences between paragraph proofs and two-column proofs.